Subscription Shares in Early 18th Century British Finance

The Share Issue vs. Subscription Share Issue Decision as a Choice between Fully-Paid Shares and Warrants

A Model based upon Asymmetric Information in Risk with special Reference to the Re-financing of the Royal African Company during the South Sea Bubble of 1720

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Summary:

i) Subscription share finance was the dominant means by which initial issues and secondary issues of equity were achieved in 18th Century and early 19th Century Britain. Can a model of capital structure choice shed light on why subscription issues were preferred in early modern British equity finance? Ever since Modigliani & Miller, we know that particular capital structure choices have to be explained in terms of the particular market inefficiencies that prompt the choices.

ii) With less-than-perfect means to enforce the terms of subscription, subscription shares need to be viewed as warrants.

iii) We model the choice between issuing fully-paid shares and subscription shares with special reference to the actual conditions of RAC finances in 1720:
    a) The firm’s existing business was moribund and the value of its debt was highly degraded;
    b) The planned re-financing of the firm was framed by a group of individuals from outside the firm, who proposed to relieve the firm of its burden of debt and to revive the business in a new business plan of its devising;
    c) The outside investors would take over the majority ownership of the firm.

iv) Our model suggests that outside investors would prefer to purchase subscription shares in preference to the purchase of fully-paid shares if they (or their agents) were possessed of superior information with regard to the riskiness of the new business plan. As information about the true risk nature of the new business plan becomes clear to the markets, outside investor wealth is more favourably affected than is original shareholder wealth. Elements of our modelling strategy are similar to those pioneered by Myers & Majluf (JoFE, 1984).

v) Our model is capable of producing a number of the observed characteristics of the RAC re-financing of 1720 described by Shea (2011):
    a) Majority ownership in the firm by new investors is determined by the need to achieve debt relief for original shareholders; it was a feature of the re-financing that would have appeared even if the re-financing was achieved by the issue of fully-paid shares.
    b) The model’s equilibrium produces a preference for subscription shares in which approximately 25 percent of the total subscription is downpayment and the remaining 75 percent is delayed. These proportions conform roughly to those of the actual RAC subscription.
c) Subscription share issues, if preferred, would be sold at issue prices that would be at a deep discount relative to the prices of fully-paid original shares. This was indeed the case in 1720.

Modelling Assumptions:

i) The firm has Assets-in-place (AIP), which are the assets of its existing business.

ii) New investors have a business plan (BP) with a fixed investment cost (INVEST).

iii) Let \( A = \text{PV}(\text{AIP}) \) and \( E(A) = \text{expected future value of AIP} \). Let \( \text{NPV} = \text{PV}(\text{BP}) - \text{INVEST} \) and \( E(\text{NPV}) = \text{expected future value of NVP} \).

iv) There are 3 distinct Risk-states. Risk-state \( i \) is defined as:

\[
\begin{bmatrix}
\text{a}_u \\
\text{npv}_u \\
\text{d}_u \\
\end{bmatrix}
\]

1) ignorance (\( i = \text{IG} \));
2) High-risk (\( i = \text{HR} \)) and
3) Low-risk (\( i = \text{LR} \)).

v) Within each risk-state, the future values (FVs) of AIP and NPV, \( a \) and npv, are perfectly correlated with each other across two possible outcome-states (up and down). With this assumption we avoid incomplete-market complications when we come to warrant pricing.

vi) \( E(A) \) and \( E(\text{NPV}) \) respectively have the same values in all risk-states. With this assumption the respective PVs of A and NPV are functions solely of the riskinesses (spreads) of their respective FVs across the outcome-states.

vii) Time and Information. In real-time, the issue decision is made in the present and the investment decision is made in the future. Prior to issue, all parties perceive risk in ignorance of which Risk-state prevails (\( i = \text{IG} \)). At the time of issue, only the controlling AGENTS know the true Risk-state, \( i = \text{HR} \) or \( \text{LR} \). At some time after issue, but well before the future investment decision must be made, the true Risk-state (HR or IR) is revealed to all parties (the Market).

viii) In Whose Interests? The controlling AGENTS decide, at time of issue, the characteristics of the issue. AGENTS can be acting in the interests of either the original shareholders or the new investors or a combination of the two.

ix) AGENTS choose K. AGENTS decide the scale (SUBSCRIPTION) and structure of the subscription. The subscription consists of a DOWNPAYMENT and a future payment, K. Whenever setting K to zero, AGENTS are implicitly deciding to issue fully-paid shares. The subscription scale can be described in either present-value or future-value terms. We chose the latter s.t. \( \text{SUBSCRIPTION} = \text{DOWNPAYMENT} \times (1 + r_f) + K \), where \( r_f \) is the assumed, commonly available risk-free rate. We refer to the FV of the DOWNPAYMENT as CASH = \( \text{DOWNPAYMENT} \times (1 + r_f) \).

x) The Market, subject to \( i = \text{IG} \), will require that DOWNPAYMENT will be a fair value to pay for the subscription. The Market will accordingly expect (require) that the issue price of new shares (\( W \)) and the number chosen to be issued (\( m \)) will be set s.t. \( mW = \text{DOWNPAYMENT} \).

xi) The firm has a DEBT, which is supposed to be repaid in the future, but may not be so under limited liability. (We will consider here only situations in which DEBT is risky s.t. \( E(A) < \text{DEBT} \). In other words, there may be at least partial default on DEBT unless positive-NPV
investment is achieved. Full default may still be avoided even if the investment decision is not made if CASH, along with E(A), is sufficiently large.)

xii) The firm’s existing (original) equity is organised into n shares. New shares will be equal to original shares once the SUBSCRIPTION is fully paid whenever whether this is done either at issue or later with the payment of K > 0. The PV of an original share is \( P_{\text{original}} \).

Comparisons with Myers & Majluf (1984) modelling:

a) M&M conflate the issue and invest decisions into one decision. In our model the decisions are separate.

b) The issue decision is actually a decision about the scale and form of new equity issues. In M&M, the scale of the issue is equal to the cost of investment and there is no pre-existing and possibly risky debt.

c) In M&M the information asymmetry embues agents with foreknowledge of the values of AIP and BP. In our model agents only have foreknowledge of the riskinesses of AIP and BP.

d) In both models positive NVP investments may be foregone. In neither model the “no investment” decision is not necessarily associated with a negative npv investment outcome. But the mechanisms of the “no investment” decision are possibly different in the two models. In M&M the decision to forego a positive npv investment opportunity may result because it will signal to the Market a value-state, which the Market will react to so that original shareholder wealth is degraded. M&M agents want to avoid such a signal. In our model the “no invest” decision will not result because agents are trying to avoid a potentially harmful Market reaction to a signal, but simply because new investors find themselves in an outcome-state in which they do not have a large enough share in the equity ownership of the firm to benefit from a positive-npv investment decision.

Further assumptions for the numerical example:

xiii) Let \( a_d = 50 \) and \( a_u = 100 \) in all risk-states.

xiv) Let the probability of being in either HR or LR be \( \frac{1}{2} \). Thus \( E(A) = 75 \). Let \( \text{DEBT} = 100 \), so that the DEBT is risky.

xv) Let \( E(NPV) = 125 \) and choose \( \text{npv}_d \) for each risk-state so that:

\[
\begin{align*}
\text{Risk - state}_{\text{IG}} & = \begin{bmatrix} 50 & 100 \\ 0 & \text{npv}_u_{\text{IG}} \end{bmatrix} \\
\text{Risk - state}_{\text{HR}} & = \begin{bmatrix} 50 & 100 \\ -90 & \text{npv}_u_{\text{HR}} \end{bmatrix} \\
\text{Risk - state}_{\text{LR}} & = \begin{bmatrix} 50 & 100 \\ 90 & \text{npv}_u_{\text{LR}} \end{bmatrix}
\end{align*}
\]

xvi) Let \( \text{NPV}_{\text{HR}} = 10 \) and \( \text{NPV}_{\text{LR}} = 100 \), so that \( \text{NPV}_{\text{IG}} = 55 \).

xvii) Let \( \text{INVEST} \) be large w/r to \( \text{DEBT} \) (100), so set \( \text{INVEST} = 200 \).
xviii) We let the riskiness of NPV determine the rest of the PVs and FVs in the model. Within any risk-state the risk-neutral probability \((p)\) consistent with NPV via the solution to:

\[
\begin{align*}
\text{NPV} &= p \cdot npv_d + (1-p) \cdot npv_u \\
\text{s.t. NPV}(1+\tau) &= p \cdot npv_d + (1-p) \cdot npv_u
\end{align*}
\]

We can now compute, via risk-neutral pricing, PVs of the subscription, original shares and debt.

xix) In anticipation of any default on the DEBT, we have to anticipate how original shareholders and new investors are to share in the settlement of the DEBT. In the RAC case there was created a trust into which subscriptions were to be paid. If the subscription failed, the trust fund may not be sufficiently large to pay the DEBT. How was the residual liability to be shared? It is not clear from the documentary evidence I have found. Assume that the liability is shared equally and define

\[
\begin{align*}
Z_d &= a_d + npv_d + \text{INVEST} - \max[\text{DEBT} + \text{INVEST} - \text{SUBSCRIPTION}] \\
Z_u &= a_u + npv_u + \text{INVEST} - \max[\text{DEBT} + \text{INVEST} - \text{SUBSCRIPTION}]
\end{align*}
\]

The Zs are respective Net Asset Values \((u,d)\) contingent upon Investment being undertaken. They take into account the possibility that new investors might well contribute enough to the firm to undertake the investment, but not enough to repay the DEBT. It was not clear from historical records whether original shareholders had an ironclad guarantee in the subscription contract that would prevent the new investors from doing this.
where $\eta = m / (m + n)$, new investors’ share in Equity if K is paid.

\[
\text{DOWNPAYMENT} = mW
\]

\[
\max[ \eta Z_d - K , 0 ] \quad \text{max}[ \eta Z_u - K , 0 ]
\]

\[
\text{nP}_{\text{original}}
\]

\[
\max[ I_d \times ((1 - \eta)Z_d + \max[\text{SUBSCRIPTION} - \text{DEBT} - \text{Invest} , 0]) , 0] \\
+ (1 - I_d) \times \max[a_d + \text{CASH} - \text{DEBT} , 0]
\]

\[
\max[ I_u \times ((1 - \eta)Z_u + \max[\text{SUBSCRIPTION} - \text{DEBT} - \text{Invest} , 0]) , 0] \\
+ (1 - I_u) \times \max[a_u + \text{CASH} - \text{DEBT} , 0]
\]

where $I_{d,u} = 1$ if Investment in down(up)-state, 0 otherwise and where $\text{CASH} = \text{DOWNPAYMENT} \times (1 + r)$. 
\[ PV(\text{DEBT}) = \frac{d \times \min\{ (1 - \eta)Z_d + \text{SUBSCRIPTION} - \text{Invest, DEBT} \} + (1 - I_d) \times \min\{ a_d + \text{CASH, DEBT} \} }{\text{ }} \]

\[ I_u \times \min\{ (1 - \alpha)Z_u + \text{SUBSCRIPTION} - \text{Invest, DEBT} \} + (1 - I_u) \times \min\{ a_u + \text{CASH, DEBT} \} , \]
PV (red) and P_original (blue) - Market-determined, in Risk-state Ignorance

PV Intrinsic W (red) and P_original (blue) in High-Risk State - Market-determined in Ignorance

PV Intrinsic W (red) and P_original (blue) in Low-Risk State - Market-determined in Ignorance
xx) The Market-determined value of $m$ (hence, $\eta$) is used to compute the Intrinsic PV Subscription, PV Original and PV DEBT. Intrinsic value is a PV known to the AGENT; that is, it uses Market-determined $m$ ($\eta$) and knowledge of the true risk-state.

xxi) $\Delta PV$ indicates the difference between Intrinsic value and Market value. AGENTS maximise $\Delta PV$. In addition, we can think of $\Delta PV$ as the change in value in Market value in Information time. As the Market proceeds from Ignorance to knowledge of the risk-state. PVs will change by $\Delta PV$.

xxii) As in M&M, AGENTS’ actions could potentially signal to the Market the true risk-state at $t=0$. In such instances all $\Delta PV$s would collapse to zero and AGENTS will have defeated their own purpose. AGENTS will therefore will make choices that result in only stable equilibria. Amongst the stable equilibria, we may find the most preferred equilibrium.

xxiii) AGENTS prefer $K>0$ regardless if they act in Original Shareholders’ interests or in the interests of Subscribers.

xxiv) The most preferred equilibrium for Subscribers does not depend upon their AGENTS’ knowledge of the true risk-state. (A bit counter to first intuition.)

xxv) Original Shareholders would prefer even higher $K$ than Subscribers. This is because with highly degraded DEBT, Original Shareholders have to hand over such a high proportion ($\eta$) of the firm to Subscribers, that only potentially very high up-sides benefit them (Original Shareholders); at any $K$, except for very high $K$, creditors receive very good relief – only at very high $K$ is there a possibility that subscriptions will not be fulfilled and original shareholders benefit from the resulting degradation in debt values.

xxvi) At equilibrium $K$, issue price of the Subscription ($W$) has to appear to be at a deep discount to the price of Original Shares.

xxvii) With regard to the RAC case – a) a large and highly value-degraded DEBT matters to the solution; b) all parties would prefer subscription shares with high $K$; c) original shareholders would prefer it, bit only mildly; d) outside Subscribers would hugely prefer to buy subscription shares rather than fully paid shares. Broadly consistent with the behaviour of Team-Chandos.

xxviii) In the general 18th C case, such as for canals, this modelling would be different. The firms would be start-ups (no assets, except for growth opportunities) and would have no large debt-overhangs. This is a bit trickier than the RAC-case, and we are working on it.

xxix) Alternative inefficient market models also beckon - supply-side constraints on capital, book-keeping costs of subscription shares versus fully-paid shares and agency-behaviour models in which governance mechanisms are different with subscription share issues from what they are when fully-paid shares are issued.